

Observation of Four-Photon de Broglie Wavelength by State Projection Measurement

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A measurement process is constructed to project an arbitrary two-mode N -photon state to a maximally entangled N -photon state (the $NOON$ -state). The result of this projection measurement shows a typical interference fringe with an N -photon de Broglie wavelength. For an experimental demonstration, this measurement process is applied to a four-photon superposition state from two perpendicularly oriented type-I parametric down-conversion processes. Generalization to arbitrary N -photon states projection measurement can be easily made and may have wide applications in quantum information. As an example, we formulate it for precision phase measurement.

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It has now been well established that only nonclassical states of light such as squeezed states can have accuracy in precision phase measurement better than the standard quantum limit which goes as $1/\sqrt{N}$ for an average photon number of N of the phase probing field [1, 2, 3]. It was argued [4] that there exists an ultimate limit, i.e., the Heisenberg limit [5] in precision phase measurement for an arbitrary state of the probe light. The Heisenberg limit goes as $1/N$ for an average photon number of N . A number of quantum states [4, 6, 7, 8, 9, 10] have been identified that can achieve such a limit when used to probe a phase shift. Among them, the $NOON$ -state of a two-mode maximally entangled N -photon superposition state has been in the forefront of discussions recently [11, 12, 13, 14, 15, 16, 17, 18]. Such a state is described as $|NOON\rangle = (|N, 0\rangle - |0, N\rangle)/\sqrt{2}$. An N -photon coincidence measurement in the superposition of the two modes gives a dependence of $1 - \cos N\varphi$ on the single photon phase shift φ . This phase dependence is typical of a fringe pattern with an N -photon de Broglie wavelength and can be managed to achieve the Heisenberg limit in accuracy in the measurement of the single-photon phase shift φ .

Because of the difficulty in preparing multi-particle entanglement, only up to four-photon de Broglie wavelength has been demonstrated so far [15, 16] through some special interference effect to cancel the unwanted states of $|N-1, 1\rangle, |N-2, 2\rangle, \dots$ etc. A number of schemes [19, 20, 21] have been proposed. Among them, the proposal by Hofmann [19] can be generalized to an arbitrary N and was demonstrated recently by Mitchell *et al.* [16] for $N = 3$ case. It is worth noting that most of the schemes are based on an N -photon coincidence measurement to discriminate against the states of photon number $< N$.

In this letter, we will approach this problem from a

completely different direction, namely, the measurement process. We will describe an interference scheme similar to that of Hofmann [19] to form an $NOON$ state projection measurement. The scheme only depends on the contribution from $NOON$ state while discarding all other orthogonal states in an arbitrary N -photon state. We demonstrate our projection method experimentally with a four-photon superposition state from two perpendicularly oriented type-I parametric down-conversion processes. Although the quantum state is not a $NOON$ state, the projection measurement allows us to demonstrate an interference fringe pattern with the typical four-photon de Broglie wavelength. Furthermore, our projection method can be easily generalized to an arbitrary N -photon state.

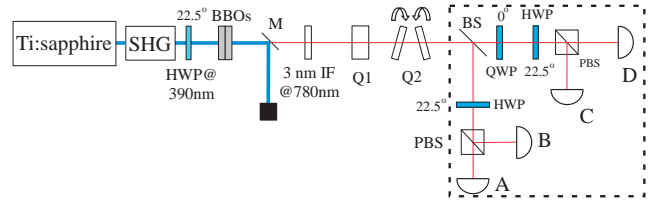


FIG. 1: Layout for four-photon $NOON$ state projection measurement with parametric down conversion.

The four-photon $NOON$ state projection measurement is depicted in the detection part of Fig.1 (inside the dotted box). The operators of the four detectors are related to the horizontal and vertical components of the input field as $\hat{b}_A = (\hat{a}_H - \hat{a}_V)/2 + \hat{b}_{A0}$, $\hat{b}_B = (\hat{a}_H + \hat{a}_V)/2 + \hat{b}_{B0}$, $\hat{b}_C = (\hat{a}_H - i\hat{a}_V)/2 + \hat{b}_{C0}$, $\hat{b}_D = (\hat{a}_H + i\hat{a}_V)/2 + \hat{b}_{D0}$. Here \hat{b}_{n0} ($n = A, B, C, D$) are some operators related to the vacuum modes $\hat{a}_{0H,V}$ in the unused beam splitter input port and make no contribution to photon detection. The four-photon coincidence rate of detectors A, B, C, D is then proportional to

$$P_4 = \langle \Phi_4 | \hat{b}_D^\dagger \hat{b}_C^\dagger \hat{b}_B^\dagger \hat{b}_A^\dagger \hat{b}_A \hat{b}_B \hat{b}_C \hat{b}_D | \Phi_4 \rangle$$

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$$= |\langle 0 | \hat{a}_H^4 - \hat{a}_V^4 | \Phi_4 \rangle|^2 / 2^8 = \frac{3}{16} |\langle 4 | NOON | \Phi_4 \rangle|^2. \quad (1)$$

Here $|\Phi_4\rangle = \sum_k c_k |N-k, k\rangle$ is a four-photon state.

To experimentally test the projection measurement scheme, we apply it to a quantum state produced from two identical but perpendicularly oriented Type-I parametric down-conversion processes. The Hamiltonian for the production of such a state has the form of

$$\hat{H} = i\hbar\chi(\hat{a}_H^{\dagger 2} + \hat{a}_V^{\dagger 2}) + H.c. \quad (2)$$

The four-photon part of the quantum state for weak interaction thus has the form of

$$|\Phi_4\rangle = \frac{\eta^2}{2} (\hat{a}_H^{\dagger 2} + \hat{a}_V^{\dagger 2})^2 |vac\rangle \\ \propto \sqrt{\frac{3}{8}} (|4, 0\rangle + |0, 4\rangle) + \frac{1}{2} |2, 2\rangle. \quad (3)$$

The *NOON* state projection gives

$$P_4 \propto 1 - \cos 4\varphi, \quad (4)$$

where φ is the single photon phase difference between the H and V polarizations. This shows the typical fringe pattern with a four-photon de Broglie wavelength.

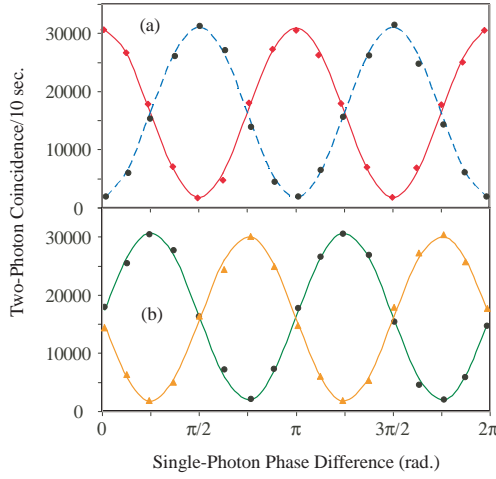


FIG. 2: Two-photon coincidence as a function of single-photon phase difference φ . (a) Coincidences between AB (circle) and between CD (diamond). The continuous curves are the least-square fit to the function $N_{2c0}(1 \pm v_2 \cos 2\varphi)$. (b) Coincidences between AD (circle) and between AC (triangle). The continuous curves are the least-square fit to the function $N_{2c0}(1 \pm v_2 \sin 2\varphi)$.

Experimental arrangement is sketched in Fig.1. A Coherent Mira Ti:sapphire laser with 150 fs pulse width and 76 MHz repetition rate is frequency-doubled to 390 nm. The harmonic field of 200 mW serves as the pump field for parametric down-conversion in two 2-mm thick BBO crystals cut for Type-I process. The polarization of

the pump field is rotated by a half wave plate (HWP) to 45° and the two crystals are oriented so that their fast (or slow) axes are perpendicular to each other. Thus the horizontal polarization of the pump field is the pump for one crystal and the vertical polarization for another crystal. The quantum state from the two crystals has the form of Eq.(3). The down-converted light first passes through an interference filter centered at 780 nm with a bandwidth of 3 nm. Then stacks of quartz plates (Q1) are used to compensate the delay between H and V polarizations of the down-converted photons. Another set (Q2) of quartz plates are used for precision phase control between H and V polarizations. The compensated light is directed to the *NOON* state projection measurement assembly. Fig.2a shows the two-photon coincidence N_c in 10 seconds between detectors A and B and between detectors C and D as a function of single-photon phase difference φ . Fig.2b presents data for coincidence measurement between A and C and between A and D. Coincidence data between B and D is nearly identical to that between A and C and coincidence between C and B is same as that between A and D. These two are not plotted. $N_c^{(AB)}$ and $N_c^{(CD)}$ are least-square-fitted to $N_{c0}(1 \pm v_2 \cos 2\varphi)$ and $N_c^{(AC)}$ and $N_c^{(AD)}$ to $N_{c0}(1 \pm v_2 \sin 2\varphi)$, respectively. They have an average visibility of $v_2 = 0.88$ after background subtraction. The four-photon coincidence counts in 5 minutes are plotted in Fig.3(a) as a function of single-photon phase difference. The data is after background subtraction. The continuous curve is a least-square fit of the discrete experimental data to $N_{4c0}(1 - V_4 \cos 4\varphi)$ with a visibility of $V_4 = 0.57 \pm 0.05$.

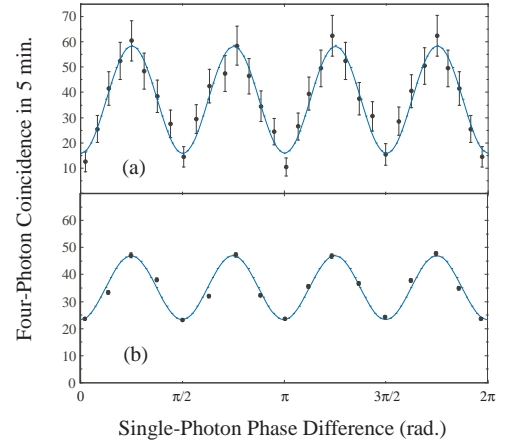


FIG. 3: Four-photon coincidence as a function of single-photon phase difference φ . The solid curves are a least square fit to the function $N_{4c0}(1 - V_4 \cos 4\varphi)$. (a) Measured four-photon coincidence with $V_4 = 0.57 \pm 0.05$ and $N_{4c0} = 37 \pm 1$. (b) Indirectly measured four-photon coincidence for 2×2 case with $V_4^{(0)} = 0.34 \pm 0.01$ and $N_{4c0} = 35 \pm 1$.

Two observations can be made from Fig.3(a). Firstly, it indeed gives the 4φ dependence on the phase as predicted by Eq.(4). Secondly, the visibility of the sinusoidal

modulation is only 57%, far short of the 100% predicted in Eq.(4). Part of the explanation of this discrepancy is from the less than perfect spatial mode match, which already results in only 88% visibility in the two-photon interference curves in Fig.2. This mode match problem arises from the walk-off between the ordinary and the extra-ordinary rays in the crystals. Because of this, the spatial mode is elongated in one direction for the first crystal but in a perpendicular direction for the second crystal. We may reduce this detrimental effect by using a mildly focussed pump beam.

However, most of the reduction in visibility actually comes from temporal mode match [22, 23, 24, 25]. It is well known that the Hamiltonian in Eq.(2) produces a pair of entangled photons in a state of $|\Phi^+\rangle = (|2H\rangle + e^{2i\varphi}|2V\rangle)/\sqrt{2}$. Four photons can be generated from two-pair production. But it is not guaranteed that the two pairs are always generated in the same time. For short pump duration, the pairs are overlapping and indistinguishable from each other, this situation is generally known as 4×1 case and the quantum state is given by Eq.(3) with the *NOON* state projection measurement result given in Eq.(4). On the other hand, when the pump duration is long, the two pairs are produced mostly at different times and are independent of each other. This is the 2×2 case with a quantum state described by $|\Phi_{2 \times 2}\rangle = |\Phi^+\rangle_1 \otimes |\Phi^+\rangle_2$. Here 1 and 2 denote the two distinguishable pairs. It can be shown [26] that the 2×2 case will also show an interference fringe pattern with four-photon de Broglie wavelength but the interference visibility is only $V_4^{(0)} = 3/7 = 0.43$.

The real system is actually in between the two extreme cases described above. Tsujino et al. [24] described the system as a statistical mixture of the two cases. A multi-mode approach [23, 25, 26] provides a complete account of all the mode match effects discussed here. From there, a general formula for the four-photon interference visibility [26] can be derived and has the form of

$$V_4 = 3(\mathcal{A} + 2\mathcal{E})v_2^2 / [(6 + v_2^2)\mathcal{A} + 2\mathcal{E}(3 - 2v_2)], \quad (5)$$

where \mathcal{A} is proportional to the accidental rate of two-pair production and $\mathcal{E}(\leq \mathcal{A})$ characterizes the overlap between the two pairs. When $\mathcal{E} = \mathcal{A}$, the two pairs are completely overlapping and the four photons are in an indistinguishable entangled state described by Eq.(3). This is the 4×1 case. On the other hand, when $\mathcal{E} = 0$, the two pairs are completely separated from each other and become independent. This is the 2×2 case. Substituting the observed values of $V_4 = 0.57 \pm 0.05$ and $v_2 = 0.88$ in Eq.(5) and solving for \mathcal{E}/\mathcal{A} , we obtain $\mathcal{E}/\mathcal{A} = 0.49 \pm 0.12$.

For the situation when the two pairs are independent of each other, the four-photon coincidence rate $R_4^{(0)}$ can be deduced from the measured two-photon coincidence rates among all four detectors. In this case, four-photon probability at four detectors in one pump pulse is simply the sum of all possible products of the two-photon

probabilities at one and other pair of detectors, that is,

$$P_4^{(0)} = P_{AB}P_{CD} + P_{AC}P_{BD} + P_{AD}P_{BC}, \quad (6)$$

where $P_{ij}(i, j = A, B, C, D)$ is two-photon probability in detectors i and j . In terms of coincidence rate, we have

$$R_4^{(0)} = (R_{AB}R_{CD} + R_{AC}R_{BD} + R_{AD}R_{BC})/R, \quad (7)$$

where $R = 76$ MHz is the repetition rate of pump pulses. The two-photon rates were measured and shown in Fig.2 and $R_4^{(0)}$ can then be derived from Eq.(7). Fig.3(b) shows the derived $R_4^{(0)}$ as a function of single-photon phase difference. The visibility from a least square fit has the value of $V_4^{(0)} = 0.34$, which is exactly same as that derived from Eq.(5) when we take $\mathcal{E}/\mathcal{A} = 0$ and $v_2 = 0.88$.

The value of \mathcal{E}/\mathcal{A} can be independently measured in our experiment. If we rotate the pump polarization to either H or V -direction, only one crystal will have down-conversion and from Ref.[26] we have

$$R_4 \propto \mathcal{A} + 2\mathcal{E}, \quad \text{or} \quad R_4 = R_4^{(0)}(1 + 2\mathcal{E}/\mathcal{A}), \quad (8)$$

where $R_4^{(0)}$ is the accidental four-photon coincidence rate for the 2×2 case ($\mathcal{E} = 0$). $R_4^{(0)}$ can be calculated from the measured two-photon coincidence rates among all four detectors from Eq.(7). Experimentally, when we set the pump polarization to H -direction, we observed $R_{AB} = 777/s$, $R_{CD} = 892/s$, $R_{AC} = 800/s$, $R_{DB} = 862/s$, $R_{AD} = 823/s$, $R_{CB} = 847/s$, and $R_4 = (103 \pm 10)/30\text{min}$ after back ground corrections. This gives rise to $R_4^{(0)} = 0.0274/s = 49.3/30\text{min}$ from Eq.(7). By Eq.(8), we then have $\mathcal{E}/\mathcal{A} = 0.54 \pm 0.05$. Within the error allowance, this value coincides with the value derived from the visibility consideration.

The value of \mathcal{E}/\mathcal{A} gives a measure of how indistinguishable the two pairs of photons are from each other in parametric down-conversion. It has a complicated dependence on temporal/spectral mode structure such as the pump pulse width, the down-conversion bandwidth, and optical filtering before detection. It also depends on the spatial mode structure such as the pump field focusing. Generally speaking, narrowing the pump pulse width and the crystal length to have a well defined time of pair production will increase the value of \mathcal{E}/\mathcal{A} . Frequency filtering can also enforce a good temporal mode match. However, these measures will reduce the count rate of the pairs and make the statistics of the data even poorer than present. For the spatial mode, we find that \mathcal{E}/\mathcal{A} increases when we have a tight focus of the pump. But this will cause poor spatial mode match between the two crystals and decrease the visibility v_2 of two-photon interference, leading to a reduced V_4 . The observed value of $\mathcal{E}/\mathcal{A} = 0.49 \pm 0.12$ is a trade-off among a good counting rate, a relatively high two-photon visibility, and a moderate four-photon interference visibility.

As claimed by many [4, 10, 15, 16, 17], the observation of multi-photon de Broglie wavelength may lead to im-

provement in precision phase measurement. For the current *NOON* state projection measurement, however, the probability P_N of a successful projection to the *NOON* state is a very small number if we do not use a *NOON* state. When P_N goes to zero for large N , we may not claim the Heisenberg limit even if we observed the N -photon de Broglie wavelength. This is because we need to apply the scheme $1/P_N$ times before we can have a successful projection. Thus the total photon number is N/P_N . It can be shown[27] that the best phase uncertainty for this case is

$$\Delta\varphi_m = \sqrt{(2 - P_N)/P_N}/N. \quad (9)$$

For finite P_N , this still gives $\sim 1/N$, i.e., the Heisenberg limit. But for most state available in laboratory, $P_N \sim 0$ as $N \rightarrow \infty$. For example, the state in parametric down-conversion from the Hamiltonian in Eq.(2) is given by

$$|PDC_N\rangle = \sum_{n=0}^N \frac{\sqrt{(2N-2n)!(2n)!}}{2^N(N-n)!n!} |2(N-n), 2n\rangle, \quad (10)$$

and we can easily find $P_N(PDC) \rightarrow 1/\sqrt{N}$. This leads to $\Delta\varphi_m \sim N^{-3/4}$ from Eq.(9).

On the other hand, we may choose a different projection measurement. This relies on the generalization of the *NOON* state projection measurement discussed in current paper to an arbitrary N -photon superposition state of $|\Phi_N\rangle = \sum_{n=0}^N c_n |N-n, n\rangle$. This is straightforward and is shown in Ref.[26].

With the arbitrary state projection measurement, we may choose the following strategy. For a given N -photon state $|\Phi_N\rangle$, we find its relevant orthogonal state through phase change, that is, find the minimum φ_m so that $\langle\Phi_N|\Phi_N(\varphi_m)\rangle = 0$. For the *NOON* state, for example, the orthogonal state is $|NOON+\rangle = (|N, 0\rangle + |0, N\rangle)/\sqrt{2}$

by a phase shift of $\varphi_m = \pi/N$ (from Eq.(4)). Now we prepare the incoming field in the orthogonal state $|\Phi_N(\varphi_m)\rangle$ and our $|\Phi_N\rangle$ -state projection measurement gives $P_{\Phi_N}(|\Phi_N(\varphi_m)\rangle) = |\langle\Phi_N|\Phi_N(\varphi_m)\rangle|^2 = 0$. Suppose there is a phase shift of $\delta = -\varphi_m$ between H and V to shift the $|\Phi_N(\varphi_m)\rangle$ state back to $|\Phi_N\rangle$ so that $P_{\Phi_N}(\delta) = 1$. Then by detecting an N -photon coincidence, we detect the small phase shift $\delta = \varphi_m$. For the *NOON* state, we have the Heisenberg limit with $\delta = \varphi_m = \pi/N$.

Of course, the *NOON* state is not easy to generate in laboratory but the state in Eq.(10) is available from parametric down-conversion. For this state, the minimum phase shift φ_m for its orthogonal state as $\varphi_m = 1.53\pi/N$ for large N [27]. Although this scheme is a little worse than the *NOON* state case, it indeed approaches the Heisenberg limit.

In summary, we have demonstrated experimentally the four-photon de Broglie wavelength by a *NOON* state projection measurement. Such projection measurement can be generalized to an arbitrary state. For the state from parametric down-conversion, its orthogonal state projection measurement may lead to Heisenberg limit in phase measurement.

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